

# Study of the Excess Pore Water Pressure in Two Different Clay Soil Layers

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**Abstract--** In consolidation theory the pore pressure is outlined as the dissipation over the hydrostatic pressure or the pore pressure in excess of steady state flow condition. The main goal of this study is to formulate the finite difference method by which calculate the pore pressure use MATLAB software and study the effect of load, coefficient of consolidation ( $C_v$ ) on the value of the pore pressure progressing in two different layers of clay soil. The variation of pore pressure with time within the depth of soil layers is investigated also. A parametric study for three case of load which are fill, strip and rectangle is conducted. The main conclusions of the study outlined that the pore pressure increase with increase the load, the increase more significant for fill load smaller than strip and rectangular loadings. Also, the pore pressure at threshold between two layers increase with the increase of the consolidation coefficient of the first layer ( $C_{v1}$ ) and decrease when the consolidation coefficient of the second layer ( $C_{v2}$ ) increase and the pore pressure decrease gradually with time in the first and second layers but it increase at the begin of applying load and then decrease until vanishes at the threshold between two layers.

**Keywords--** Soil Consolidation; Effective Stress; Excess Pore Water Pressure

## I. INTRODUCTION

When a layer of soil compressed by applying load during the construction it will present an amount of compression. This compression is accomplished by many ways, such as rearrange of soil particles or bulge of pore water. A decrease of moisture content of saturated soil without replacement of the water by air is named consolidation. When saturated clay which have a low permeability subjected to a compression resulting by loading, the pore water pressure instantaneously increase with a time retardation between the load application and the bulge of the pore water and, thus, the settlement. One of the procedures to calculate the one-dimensional consolidation settlement is the finite difference solution which is adopted in this study.

Reference [1] awarded a solution for soil consolidation analysis for vertical drains under ramp load. The summary of study founded that in the note that the average degree of consolidation,  $U$ , in terms of the time factor  $T$  considered to be approximately independent of the parameters  $N$  and  $L$ , presenting good normalization behavior.

Reference [2] performed a nonlinear theory for sand drain consolidation of clayey soils under time dependent loading. The procedure is achieved using variables separation method. Behavior of horizontal and vertical time-loadings drainage applicable consolidation is investigated. The analytical solution of consolidation showed that the degree of consolidation can be determined either depending on settlement ( $U_s$ ) or depending on effective stress ( $U_p$ ). Also the former indicated that the settlement development rate, the last shows that the rate of the increase of effective pressure or the rate of the rout of pore pressure and for the like parameters,  $U_p$  is always less than  $U_s$ .

Reference [3] presented a double layered consolidation analytical solution for embankment constructed on footing moderately imbedded penetrated by deep mixed columns, one way and two way drainage are conducted. The calculation of time-settlement relationship used consolidation algorithm. Laplace method of transformation was adopted to solve the consolidation equations and Stehfest algorithm was adopted to explain the inverse Laplace transform for time-dependent loading. Two stage of loading describes an effect of the first loading time on the consolidation degree of the system. The study shows that for the double layered soil system, the stiffness have an significant role in the consolidation role and the hard soil exhibit faster rate of consolidation.

Reference [4] adopted a (FDM) to present numerical solutions for pore water pressure and average consolidation degree. The modification of theory of one-dimensional consolidation is modified based on Non-Darcian flow produced by non-Newtonian liquid to consider the variation in the total vertical stress with depth and time. The study founded that the water flow law parameters have an important effect on the rate of consolidation and the percent of the equivalent water head of external load to the soil layer depth have a strong influences on the rate of consolidation. Also, the rate of consolidation is slow when the applying load is slow.

Reference [5] used a (FEM) to estimate the rate of frozen thawing soils. Abaqus FEA was used to formulate layers of pavement with a sinusoidal temperature of surface and compared with the analytical solution depending on Stephan's method which is assumed a constant surface temperature. The results of study founded that for the frozen pavements layers, a steady thawing rate is obtain. A higher rate of thawing in low permeability frozen soils founded in high pore pressure and the thawing of the late spring be augured from the change in temperature of pavement from available climatic data, and physical and thermal features of the pavements materials.

## II. NUMERICAL SOLUTION

The solution of consolidation in layered soils is not forever perform by a closed form solution because of there are many parameters involved such as permeability coefficients ( $k_i$ ), the layers thickness ( $H_i$ ), and consolidation coefficients ( $C_{vi}$ ). Fig. 1 shows the nature of the degree of consolidation of a two layered soil. Numerical solutions is a good approach.

The excess pore water pressure at any point using finite difference method can be calculated by (1) below, [6].

$$\bar{u}_{o,\bar{t}+\Delta\bar{t}} = \frac{\Delta\bar{t}}{(\Delta\bar{z})^2} (2\bar{u}_{1,\bar{t}} - 2\bar{u}_{o,\bar{t}}) + \bar{u}_{o,\bar{t}} \quad (1)$$

The excess pore water pressure at the interface of two different types of clayey soils was involved and derived in (2) below:

$$k \frac{\partial^2 u}{\partial z^2} = \frac{1}{2} \left[ \frac{k_1}{(\Delta z)^2} + \frac{k_2}{(\Delta z)^2} \right] \times \left( \frac{2k_1}{k_1 + k_2} u_{1,t} + \frac{2k_2}{k_1 + k_2} u_{3,t} - 2u_{o,t} \right) \quad (2)$$

Where  $k_1$  and  $k_2$  are the coefficients of permeability in layers 1 and 2, respectively, and  $u_{o,t}$ ,  $u_{1,t}$ , and  $u_{3,t}$  are the excess pore water pressures at time  $t$  for points 0, 1, and 3, respectively.

Also, the average volume change for the element at the boundary is:

$$\frac{k}{C_v} \frac{\partial u}{\partial t} = \frac{1}{2} \left( \frac{k_1}{C_{v1}} + \frac{k_2}{C_{v2}} \right) \frac{1}{\Delta t} (u_{o,t+\Delta t} - u_{o,t}) \quad (3)$$

Where  $u_{o,t}$  and  $u_{o,t+\Delta t}$  are the excess pore water pressures at point 0 at times  $t$  and  $t+\Delta t$ , respectively. Equating the right-hand sides of (2) and (3), then

$$\left( \frac{k_1}{C_{v1}} + \frac{k_2}{C_{v2}} \right) \frac{1}{\Delta t} (u_{o,t+\Delta t} - u_{o,t}) = \frac{1}{(\Delta z)^2} (k_1 + k_2) \times \left( \frac{2k_1}{k_1 + k_2} u_{1,t} + \frac{2k_2}{k_1 + k_2} u_{3,t} - 2u_{o,t} \right) \quad (4)$$

Assuming  $1/t_R = C_{v1}/Z_R^2$  and depending finite difference solution, [6], we get

$$u_{o,t+\Delta t} = \frac{1 + k_2/k_1}{1 + (k_2/k_1)(C_{v1}/C_{v2})(\Delta z)^2} \frac{\Delta t}{\Delta z^2} \times \left( \frac{2k_1}{k_1 + k_2} \bar{u}_{1,t} + \frac{2k_2}{k_1 + k_2} \bar{u}_{3,t} - 2\bar{u}_{o,t} \right) + u_{o,t} \quad (5)$$

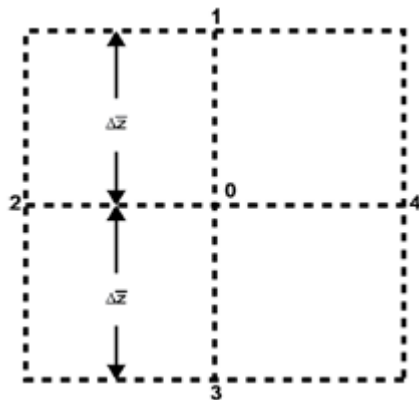


Figure 1: Numerical Solution for Consolidation

### III. CASE STUDY

Fig. 2 show the details of the case study problem adopted to make a numerical example solution for the calculation of excess pore pressure in two different clay layers.

The case study represent a strip footing with  $B = 4m$ . rests on the ground surface with a specified load.

It is required to calculate the pore water pressure distribution in clay I and II below the centerline of footing at time = 5, 10, 15, 20, 25, and 30 days .

Solution:

\*For strip footing, [6], find  $\sigma_x$ ,  $\sigma_z$ ,  $\tau_{xz}$

\*divide the tow clay layer into  $(14/2) = 7$  layers

$$\bar{u} = \frac{u}{u_R}, \bar{z} = \frac{z}{z_R}, \bar{t} = \frac{t}{t_R}$$

let

$$u_R = 1 \Rightarrow z = 14m, \bar{z} = 1$$

$$\therefore \Delta \bar{z} = \frac{\bar{z}}{\text{no. of layers}} = \frac{1}{7}$$

$$= \frac{\text{thickness of each layer}}{\text{total thickness}} = \frac{2}{14} = \frac{1}{7}$$

$$\therefore \frac{\Delta t_1}{(\Delta \bar{z}_1)^2} = \frac{c_{v1} t}{z_R^2 \times (\Delta \bar{z})^2} = 0.09t$$

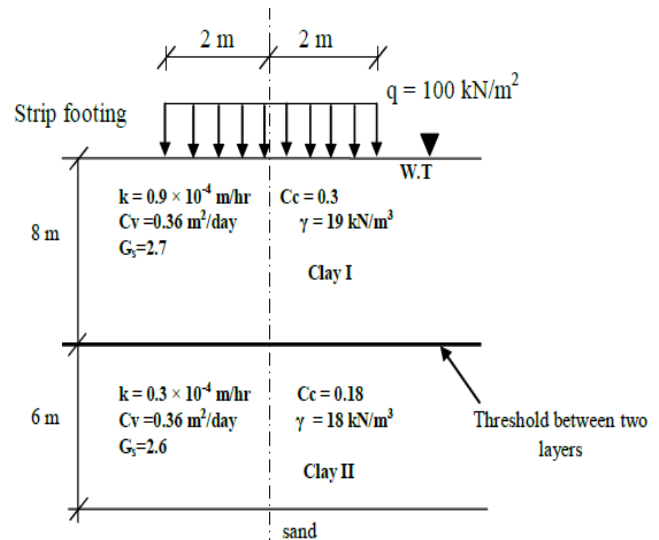


Figure 2: Case Study Problem

$$\text{at } \Delta t = 5 \text{ days} \quad \therefore \frac{\Delta t_1}{(\Delta \bar{z}_1)^2} = 0.45 < 0.5 \dots \text{ok.}$$

$$\frac{\Delta t_2}{(\Delta \bar{z}_2)^2} = \frac{c_{v2} t}{z_R^2 \times (\Delta \bar{z})^2} = 0.03t$$

$$\text{At } \Delta t = 5 \text{ days} \quad \therefore \frac{\Delta t_2}{(\Delta \bar{z}_2)^2} = 0.15 < 0.5 \dots \text{ok.}$$

if  $x=0$ ,  $\tau_{xz}=0$

If  $\tau_{xz}=0$ ,  $\sigma_z = \sigma_1$ ,  $\sigma_x = \sigma_3$ ,  $\sigma_2 = v \times (\sigma_1 + \sigma_3)$

If  $\tau_{xz} > 0$

#### IV. PARAMETRIC STUDY

##### A. MATLAB Package

To perform the parametric study, MATLAB Package is used to formulate the calculations of excess pore pressure for any case. Using MATLAB affords to the user analysis of data, developing algorithms, and creating models and applications. The language, tools, and built-in math functions enable researchers explore multiple approaches and reach a solution faster than with spreadsheets or traditional programming languages. MATLAB can be used for a range of applications, including signal processing and communications, image and video processing, control systems, test and measurement, computational finance, and computational biology, [7]. The outputs was tabulated in EXCEL package to draw a suitable and required graphs to make the parametric study comparisons.

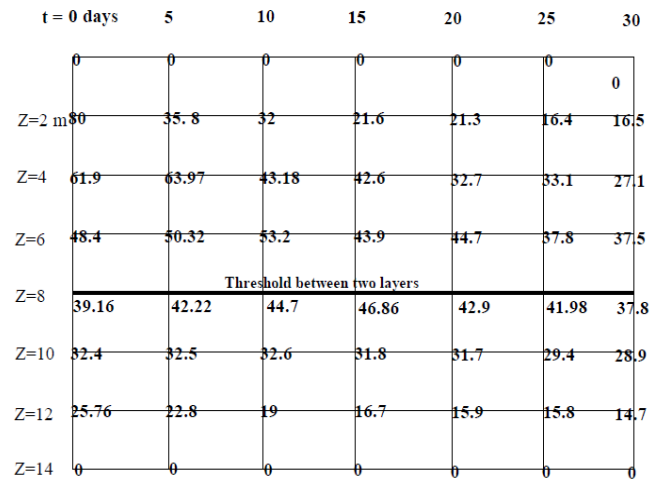


Figure 3: Mesh of Finite Difference Method Results of Case Study

##### B. Results and Discussion

In this section, the information of the case study in preceding section are used to make the parametric study. Many values for the parameters included in the calculations of the pore water pressure are investigated. The effect of applied load and coefficient of consolidation ( $C_v$ ) on the excess pore water pressure is presented. The relationship between time and the excess pore water pressure is studied then. The variation of excess pore water pressure within the soil depth is presented also, coefficient of consolidation and excess pore water pressure for different values of loads are presented. Parametric study for fill, strip and rectangular loadings are studied.

##### a. Effect of load on pore water pressure

The effect of applied load acting on the surface of soil at time of 30 days has been investigated. Figs. 4, 5 and 6 show the effect of load on the pore water pressure at the depth within the first layer ( $z = 4\text{m}$ ), threshold between two layers and within the second layer ( $z = 12\text{m}$ ) for fill, strip ( $B = 4\text{m}$ ), and rectangular loading respectively. It can be seen that the pore pressure increase with increase the load. The pore pressure increase significantly for fill load.

$$\sigma_1, \sigma_3 = \frac{\sigma_x + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_z - \sigma_x}{2}\right)^2 - \tau_{xz}^2}$$

then,

$$\Delta u = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} +$$

$$a\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

$$\bar{u}_{o,i+\Delta t} = \frac{\Delta t}{(\Delta z)^2} (\bar{u}_{1,i} + \bar{u}_{3,i} - 2\bar{u}_{o,i}) + \bar{u}_{o,i}$$

if  $k_1 \neq k_2$

$$\bar{u}_{o,i+\Delta t} = \frac{1 + \frac{k_1}{k_2}}{1 + \left(\frac{k_2}{k_1}\right)\left(\frac{c_{v1}}{c_{v2}}\right)} \times \frac{\Delta t}{(\Delta z)^2} \times$$

$$\left( \frac{2k_1}{k_1 + k_2} \bar{u}_{1,i} + \frac{2k_2}{k_1 + k_2} \bar{u}_{3,i} - 2\bar{u}_{o,i} \right)$$

$$\sigma_1, \sigma_3 = \frac{\sigma_x + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_z - \sigma_x}{2}\right)^2 - \tau_{xz}^2}$$

then,

$$\Delta u = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} +$$

$$a\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

$$\bar{u}_{o,i+\Delta t} = \frac{\Delta t}{(\Delta z)^2} (\bar{u}_{1,i} + \bar{u}_{3,i} - 2\bar{u}_{o,i}) + \bar{u}_{o,i}$$

if  $k_1 \neq k_2$

$$\bar{u}_{o,i+\Delta t} = \frac{1 + \frac{k_1}{k_2}}{1 + \left(\frac{k_2}{k_1}\right)\left(\frac{c_{v1}}{c_{v2}}\right)} \times \frac{\Delta t}{(\Delta z)^2} \times$$

$$\left( \frac{2k_1}{k_1 + k_2} \bar{u}_{1,i} + \frac{2k_2}{k_1 + k_2} \bar{u}_{3,i} - 2\bar{u}_{o,i} \right)$$

$\Delta\sigma$  is calculated according to [6].

Table 1 and Fig. 3 show the details of the outputs of the finite difference procedure shown above for the mesh of soil profile presented in the case study.

Table 1: Case Study Solution for  $\Delta u$

z	x	z/b	x/b	$\sigma_z$	$\sigma_v$	$\tau_{xz}$	$\sigma_1$	$\sigma_3$	$\sigma_2$	$\Delta u$
0	0	0	0	100	100	0	100	100	100	$\theta$ (above is open)
2	0	0.5	0	95.94	44.98	0	95.94	44.98	70.46	30
4	0	1	0	81.83	18.17	0	81.83	18.17	50	61.9
6	0	1.5	0	66.78	8.03	0	66.78	8.03	37.4	48.4
8	0	2	0	55.08	4.1	0	55.08	4.1	29.6	39.16
10	0	2.5	0	46.17	2.28	0	46.17	2.28	24.2	32.4
12	0	3	0	37.26	0.46	0	37.26	0.46	18.86	25.76
14	0	3.5	0	28.35	0	0	28.35	0	14.18	$\theta$ (under with same)

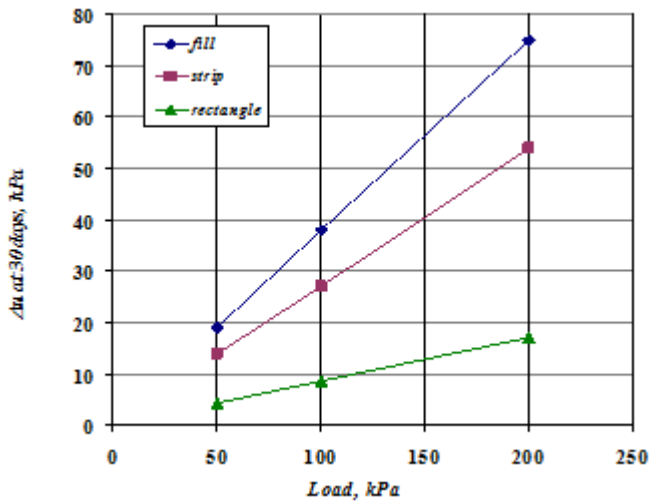


Figure 4: Effect of Load on Pore Pressure within The First Layer ( $z = 4$  m)

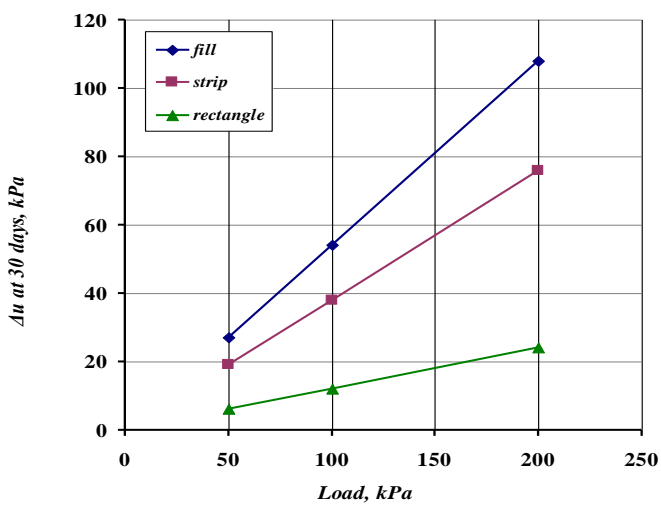


Figure 5: Effect of Load on Pore Pressure at Threshold Between Two Layers

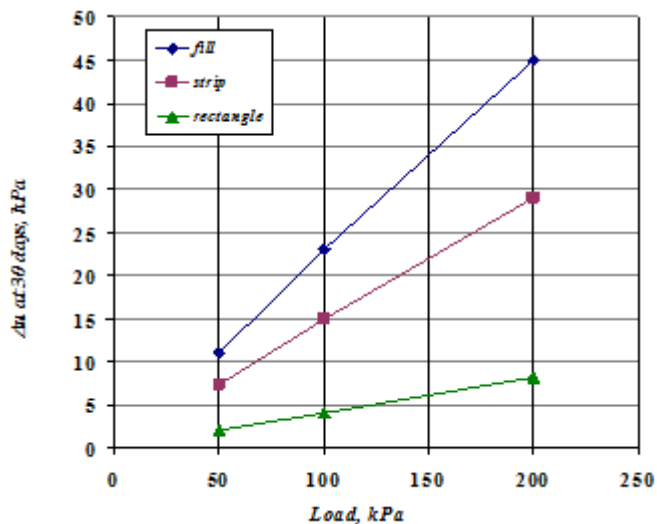


Figure 6: Effect of Load on Pore Pressure within The Second Layer ( $z = 12$  m)

#### b. Effect of coefficient of consolidation ( $C_v$ ) on pore water pressure

The effect of coefficient of consolidation ( $C_v$ ) on the pore water pressure has been investigated. Fig. 7 show the effect of coefficient of consolidation for the first layer ( $C_{v1}$ ) on the pore water pressure at threshold between two layers for constant

500 kPa.

Fig. 8 show the relationship between coefficient of consolidation for the second layer ( $C_{v2}$ ) and the pore pressure at threshold between two layers. It can be seen that the increase in  $C_{v1}$  increase the excess pore water pressure for the three types of loadings. It can be seen also that the excess pore water pressure at the end of the period of the case study (30 days) decrease clearly for the fill and strip loadings and with a little amount for rectangular loading.

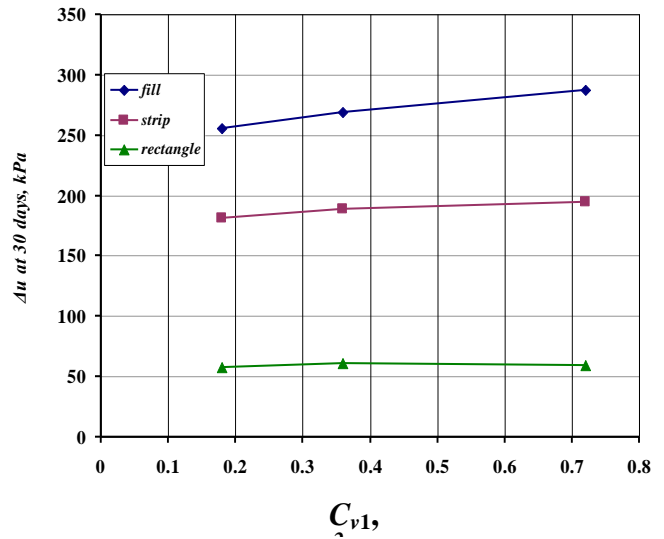


Figure 7: Effect of  $C_{v1}$  on Pore Pressure at Threshold for 500 kPa Loadings

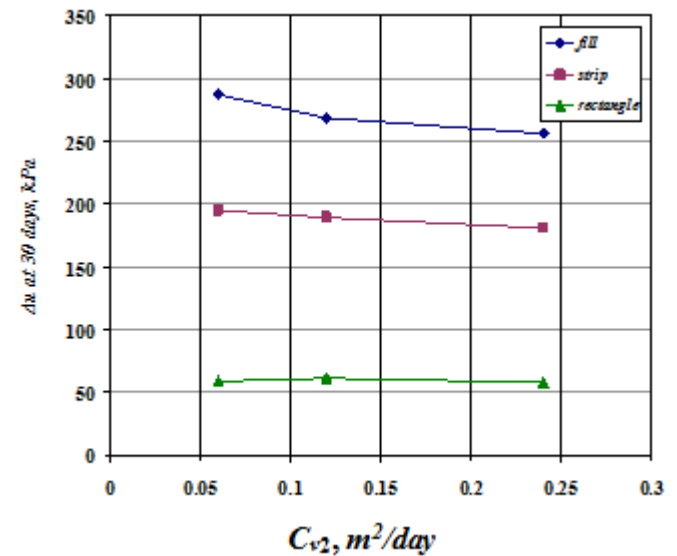


Figure 8: Effect of  $C_{v2}$  on Pore Pressure at Threshold for 500 kPa Loadings

#### c. Relationship between pore water pressure and time

Figs. 9, 10 and 11 show the relationship between the time and excess pore water pressure in first layer, threshold between two layers and second layer respectively. These cases were studied for load 100 kN/m² only. It can be seen that the pore water pressure increase within the half interval of time then it decreases, that is correct at threshold between two layers while the pore water pressure decrease gradually with time within the layers of soil ( $z = 4$  m,  $z = 12$  m). that can be recognized as the water at threshold has no free way to escape out, then the pore pressure rise for a few days then when the water drainage into two sides of layers (free drainage) the pore pressure decrease until dissipated. It can be seen also that the rectangular footing

has a little amount of effect on the second clay layer because of that the load will be vanishes through the depth, that explain the low values of pore water pressure with time for the second layer.

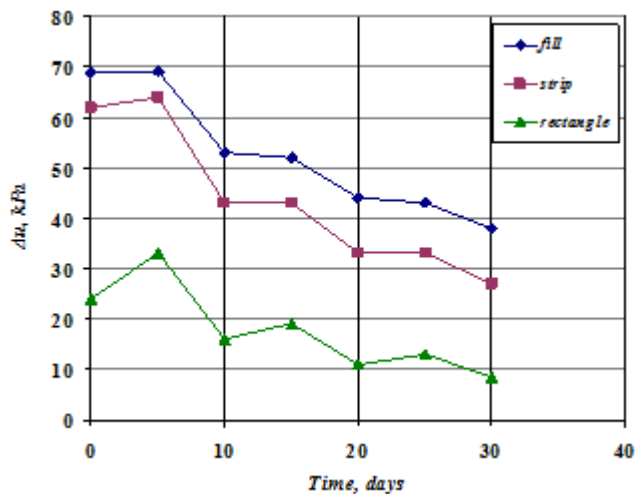


Figure 9: Relationship Between Time and Pore Pressure in the First Layer ( $z = 4m$ ) for 100 kPa Loadings

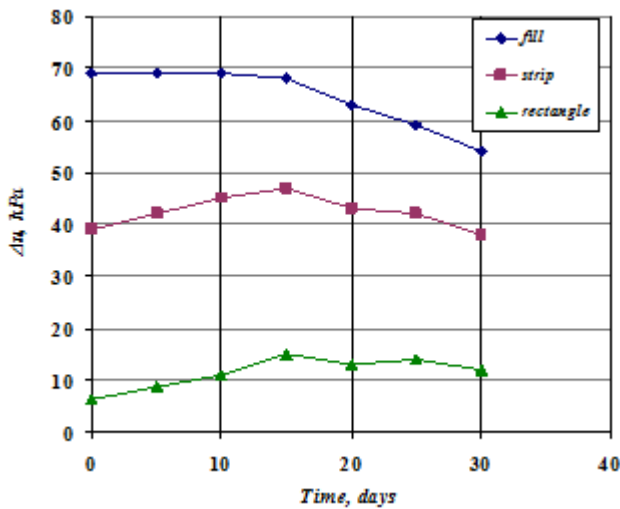


Figure 10: Relationship Between Time and Pore Pressure at the Threshold Between Two Layers for 100 kPa loadings

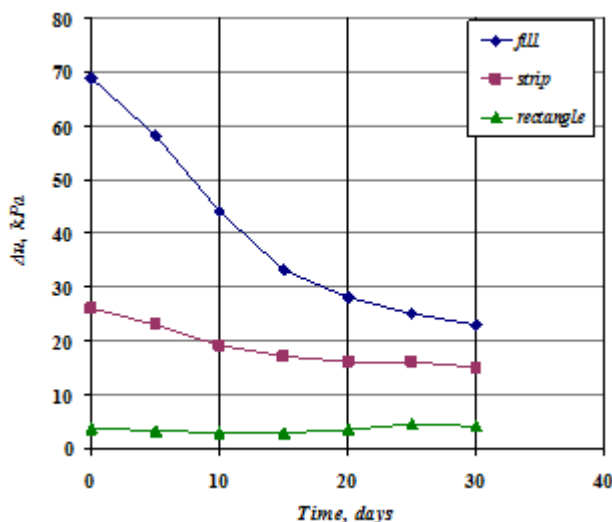


Figure 11: Relationship Between Time and Pore Pressure in the Second Layer ( $z = 12m$ ) for 100 kPa Loadings

#### d. Variation of pore water pressure within the soil layers

The variation of pore pressure with depth is shown in Figs. 12,

13 and 14. Fig. 12 deals with the variation of pore pressure for fill loading. It can be seen that the pore pressure increase gradually with depth up to the threshold between two layers, then gradually decrease until reaches to the previous layer at the end of second layer. The same behavior can be seen for the strip and rectangular loadings with some wiggle in values but generally it can be say the pore pressure increase through the layers up to the previous way at the top and bottom as in the adopted case study (Figs. 13 and 14).

It is very clear for the fill loading that the variation is the initial pore pressure constant with depth, and for strip and rectangular the variation it can be considered as triangular. That is have a highly correct for the pore water pressure at time equal to zero as the moment of load application.

## V. VERIFICATION OF RESULTS

The consolidation progressing can be visualized by drawing many curves for The consolidation can be shown by plotting many curves of  $u_e$  opposite  $z$  for many values of  $t$ . These curved lines are named isochrones and their shape depend on the primary distribution of pore pressure and drainage conditions at the thresholds of the clay layer. An open layer is the layer of both the lower and upper boundaries are free draining; a half closed layer is a layer of only one boundary is free draining. Fig. 14 show a sample of isochrones. In (a) of the graph the primary variation of  $u_i$  is steady and for an open layer of thickness  $2d$  the isochrones are elegant about the line of center. The upper half of this drawing represents the half closed layer case of thickness  $d$ . The gradient of an isochrone at any depth represents the hydraulic gradient and indicates the flow direction. In parts (b) and (c) of the drawing, the direction of flow changes over certain parts of the layer with a triangular distribution of  $u_i$ . In part (c) the lower boundary is impermeable and for a time swelling happen in the lower division of the layer [8].

The results of this study supported by the distribution of excess pore pressure presented in major text books which deals the subject of consolidation and pore pressure. Figs. 12, 13, and 14 show a significant similarity of the initial excess pore water pressure ( $\Delta u$  at  $t = 0$ ) with the cases presented by [8] in Fig. 15, so, it can be concluded that the results of study have a good credibility with the other references and text books.

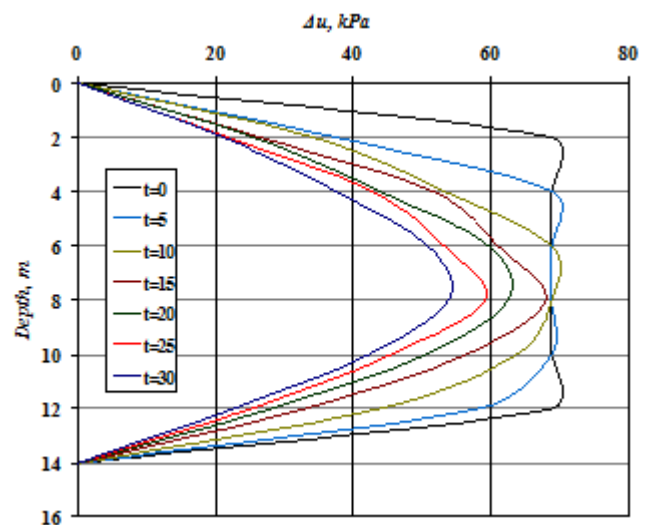


Figure 12: Variation of Pore Water Pressure with Depth for Fill Loading



## CONCLUSIONS

Study of excess pore water progress within different saturated clay soil layers for different types of the application load is presented in this paper. The formulation of problem depends on the numerical solution using finite difference method. MATLAB software package is used to perform the equations of the numerical solution for the case study and the parametric study. The following founding can be listed as:

1. The excess pore pressure increase with increase the load.
2. The excess pore pressure increase more significant for fill load smaller than strip and rectangular loadings.
3. The excess pore pressure increase mainly with the increase of load in fill rather than the strip and rectangular.
4. The pore pressure at threshold between the clay layers increase with the increase of the consolidation coefficient of the first layer ( $C_{v1}$ ) and decrease when the consolidation coefficient of the second layer ( $C_{v2}$ ) increase.
5. The pore pressure decrease gradually with time in the first and second layers but it increase at the begin of applying load and then decrease until vanishes at the threshold between two layers.
6. The variation of pore pressure at the moment of application of load is constant with depth and have a triangular shape for strip and rectangular footings.

## References

- [1] Zhu, K. & Yin, J.-H., "Consolidation of soil with vertical and horizontal drainage under ramp load", Technical Notes presented in Geotechnique 51, 2001, No. 4, , pp. 361-367.
- [2] Xueyu Geng, "Non-linear consolidation of soil with vertical and horizontal drainage under time-dependent loading", paper presented in International Conference on Advanced Computer Theory and Engineering, 2008, Australia, pp. 800-804.
- [3] Miao Linchang et al., "Consolidation of a Double-Layered Compressible Foundation Partially Penetrated by Deep Mixed Columns", paper presented in Journal of Geotechnical and Geoenvironmental Engineering, 2008, 134, pp. 1210-1214.
- [4] Xie Kanghe, Chuanxun Li, Xingwang Liu And Yulin Wang, "Analysis of one-dimensional consolidation of soft soils with non-Darcian flow caused by non-Newtonian liquid", paper presented in Journal of Rock Mechanics and Geotechnical Engineering, 2012, 4 (3), pp. 250-257.
- [5] Yesuf G.Y., Hoff I. and Vaslestad J., "Development of excess pore-water pressure in thawing process of frozen subgrade soils: Based on analytical solutions and finite element method", paper presented in 18<sup>th</sup> International Conference on Soil Mechanics and Geotechnical Engineering, 2013, Technical Committee 103, Paris, pp. 857-860.
- [6] Braja. M. Das, "Advanced Soil Mechanics", Third Edition, Taylor & Francis e-Library, 2007.
- [7] MATLAB Manual, "MATLAB R2014a 8.3.0.532 Win 32 Software", License Number 271828, 2014.
- [8] Craig R. F, "Craig's Soil Mechanics", Seventh Edition, Department of Civil Engineering, University of Dundee UK, Taylor & Francis Group, 2004.

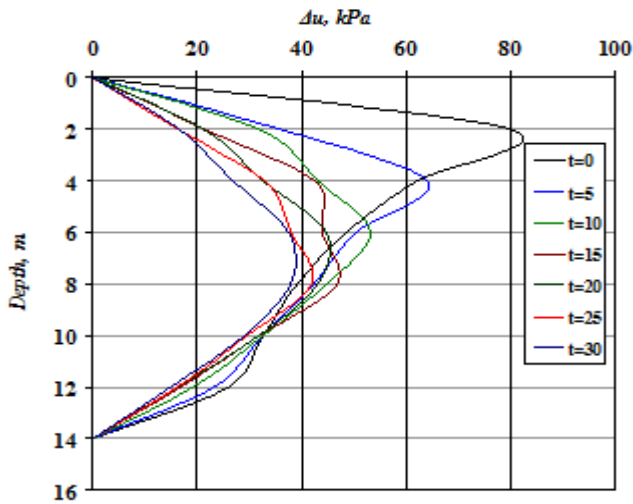


Figure 13: Variation of Pore Water Pressure with Depth for Strip Loading

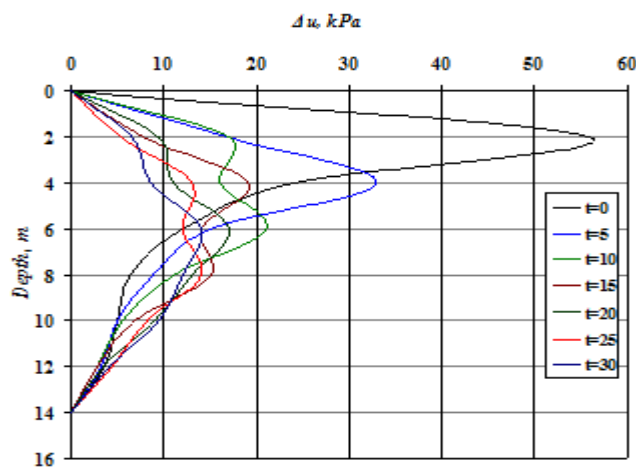


Figure 14: Variation of Pore Water Pressure With Depth for Rectangular Loading

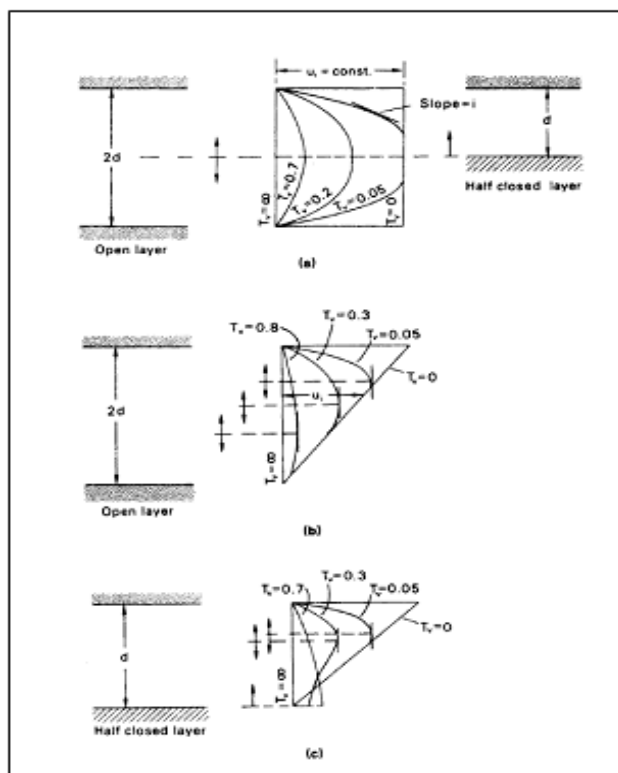


Figure 15: Initial Variations of Excess Pore Water